Any move in Blokkology can be decomposed into a horizontal and vertical component. The order of these components does not matter (analogous to the commutativity of vector addition).

Thus, letting (*j*, *k*) signify the square in the *j*th rank and *k*th file of the board:

$$\begin{gathered}(j,k) \to (p,q) \\ \sim (j,k) \to (j,q) \to (p,q) \\ \sim (j,k) \to (p,k) \to (p,q) \\ \sim [(j,k) \to (j,q)] + [(j,k) \to (p,k)]\end{gathered}$$

Thus in order to move $(j,k) \to (p,q)$, it must be possible to move both

$$[(j,k) \to (j,q)] \text{ and } [(j,k) \to (p,k)].$$

Now consider a multiset of *n* tiles $T = \{t_1, t_2, \dots\ t_n\}$, some of which may be identical, where each *t* is an integer between 1 and 9 inclusive.

We define a **1-dimensional Blokkological combination of *T*** as any sum of the form

$$M_1 t_1 + \dots + M_n t_n = \sum_{i=1}^{n} M_i t_i, \text{ where } M_i \in \{-1, 0, 1\}.$$

That is, you can move *s* squares, either horizontally or vertically, iff *s* is a 1-dimensional Blokkological combination of your tiles. (NB: the addition here is also commutative, since the order of moves in Blokkology does not affect the ending square.)

Lemma: For any *T*, $\sum_{i=1}^{n} M_i t_i$ is a 1-dimensional Blokkological combination of *T* iff $-\sum_{i=1}^{n} M_i t_i$ is also such a combination. (This is just another way of saying that if you can move *left* 4 squares, you can also move *right* four squares.)

In order to move horizontally from (*j*, *k*) to (*j*, *q*), for fixed *j*, *k*, and all *q* such that $1 \le q \le 10$, *T* must have the combination *q* – *k*. Therefore, since on the 10-by-10 Blokkology board $1 \le k \le 10$, and by our previous lemma, *T* must have as combinations the 10 distinct elements of $\mathbf{Z}_{10}$. (Same vertically.)

Now we will prove that if each of these numbers 0-9 is a 1-dimensional Blokkological combination of *T*, then $n > 2$. That is, at least three tiles are needed to span any given rank (or column) of the board.

Define *B*(*T*) as the number of non-zero 1-dimensional Blokkological combinations formed from *T*. Then $B(T) = 3^n$, since $|M| = 3$.

Therefore $B(2) = 3^2 = 9 < 10$. Therefore two tiles cannot span an entire rank or file.

(Three tiles, however, *can* span a rank or file. For example, let $T = \{1, 3, 5\}$. Then each element of $\mathbf{Z}_{10}$ can be expressed as a 1-dimensional Blokkological combination of *T*:

0 = 0, 1=1, 2=3-1, 3=3, 4=1+3, 5=5, 6=1+5, 7=5+3-1, 8=3+5, 9=1+3+5)

Thus, the horizontal component of an arbitrary move requires 3 tiles.

By symmetry, the vertical component of that arbitrary move also requires 3 tiles.

And so, since tiles cannot be re-used in a move, at least 3 + 3 = **6** tiles are required to span the entire board. QED.